Boundary shear stress measurements by two tubes

Mesure da la contrainte de cisaillement à la paroi au moyen de deux tubes

MING JIN
*National Research Council Associate,
Hydrological Research Laboratory,
National Weather Service, NOAA,
1325 East-West Highway,
Silver Spring, MD 20910, USA*

SUMMARY
A two-tube method is proposed to extend the Preston tube technique for determining the local shear stress in both rough and smooth surfaces. The analytical calibration functions of the Preston tube are derived based on the Rotta's model for the velocity profile on smooth and rough surfaces and a formula for the displacement factor of the Preston tube. The new method, unlike the conventional Preston tube method, is a combination of the measurement by two tubes of different diameters and iterative computation by the analytical calibration functions to determine the roughness parameter and local shear stress. The analytical calibration curves are compared with experimental data. The use of the two-tube method on the smooth surface is verified by experiment.

RÉSUMÉ
Une méthode à deux tubes est proposée pour étendre la technique du tube de Preston à la détermination de la contrainte de cisaillement locale, à la fois sur des surfaces rugueuses et lisses. Les relations analytiques de calibrage du tube de Preston sont dérivées du modèle de Rotta pour le profil de vitesse sur des surfaces lisses et rugueuses et du facteur du déplacement du tube de Preston. La nouvelle méthode, à la différence de la méthode classique du tube de Preston, est une combinaison des mesures au moyen de 2 tubes de diamètres différents et de calculs itératifs au moyen des relations de calibrage pour déterminer le coefficient de rugosité et la contrainte de cisaillement locale. Les courbes du calibrage analytique sont comparées aux données expérimentales. L'utilisation de la méthode à deux tubes a été vérifiée par des essais sur surface lisse.

1 Introduction

The Preston tube technique (Preston 1954; Patel 1965) is one of the most widely used shear stress measuring techniques due to its simple construction and practicability. The technique has been recognized as a convenient and reliable method for measuring the boundary shear stress in classical two-dimensional turbulent flows over smooth surfaces. Some studies have been made to extend the technique to the shear stress measurement in flows over rough surfaces (Hwang and Laursen 1963; Ghosh and Roy 1970; Hollick 1976; Hollingshead and Rajaratnam 1980).

Although the Preston tube technique can be successfully used on smooth surfaces, some problems arise when it is used on a rough surface. The additional parameters relating to the surface roughness condition make it more complicated for developing a calibration chart by both analytical and experimental methods than for a smooth surface. The studies mentioned above employed the Nikuradse's expression for velocity distribution on a rough boundary to develop calibration curves of the Preston tube. The calibration curves derived in such a way can be used only when the sand equivalent roughness height of the surface and the position of the zero velocity datum are precisely predetermined.

Revision received February 7, 1995. Open for discussion till December 31, 1995.

331A

The limitation of the method is due to not only the difficulty of determining the sand equivalent roughness height and the uncertainty of the reference datum, but also the fact that the sand equivalent roughness height may not be a suitable roughness parameter for many open channel flows.

In this paper, a two-tube method is presented with the purpose of overcoming the difficulty of extending the Preston tube technique to surfaces in the transition regime or the completely rough regime. For smooth surfaces, a two-tube method is also proposed to eliminate the necessity of measuring the static pressure simultaneously. The analytical calibration function of the Preston tube for both smooth and rough surfaces is derived based on a Rotta-Dreist model for the law of the wall and a proposed formula for the displacement factor of the Preston tube. The two-tube method is also applied on smooth surfaces in which only total pressure measurements are needed. The developed analytical calibration curves (ACC) are compared with some experimental data and the two-tube method on smooth surfaces is verified by experiments.

2 The analytical calibration function

The Preston tube technique is based on the inner layer law of turbulent boundary layer

$$u^{+} = f(y^{+}); \quad u^{+} = \frac{u}{u^{*}}, \quad y^{+} = \frac{(yu)^{*}}{\upsilon} \tag{1}$$

The law of the wall measures a quantity proportional to the velocity (the difference between the Preston and the static pressure) to correlate with the local shear stress. The correlation between the dynamic pressure and the corresponding wall shear stress involves the calibration function

$$\frac{\tau_0 d^2}{4\rho\upsilon^2} = F\left(\frac{\Delta p d^2}{4\rho\upsilon^2}\right) \tag{2}$$

in which Δp is the difference of the total pressure from the Preston tube and the static tapping readings, d is the external diameter of the Preston tube, ρ and υ are the fluid density and kinematic viscosity, respectively, and τ_0 the wall shear stress.

The calibration function F in Eq. 2, holds true only in classical two-dimensional turbulent boundary layers on smooth surfaces where the Reynolds number is the only parameter in the velocity distribution. For flows over rough surfaces, the situation is complicated by parameters describing the roughness. For the flows over a specified surface with given form and distribution of roughness, the magnitude of the roughness projection, k, characterizes the surface condition. A Preston tube with external diameter d on a rough surface with characteristic roughness height k is illustrated in Fig. 1(c). The smooth surface can be regarded as a special case when $k = 0$. Since the dynamic pressure, $p_d = p - p_0$, always corresponds with an effective wall distance, y_e, of the corresponding velocity, one can write

$$(p - p_0) = \frac{\rho}{2}(u^2)_{y = y_e}; \quad y_e = k + K_e\frac{d}{2} \tag{3}$$

$$\frac{\Delta p d^2}{4\rho\upsilon^2} = \frac{1}{2}\left(\frac{\tau_0 d^2}{4\rho\upsilon^2}\right)\left(\frac{u}{u_*}\right)^2_{y = y_e} \tag{4}$$

in which K_e is a displacement factor accounting for the deviation of the effective center of the Preston tube from its geometric center.

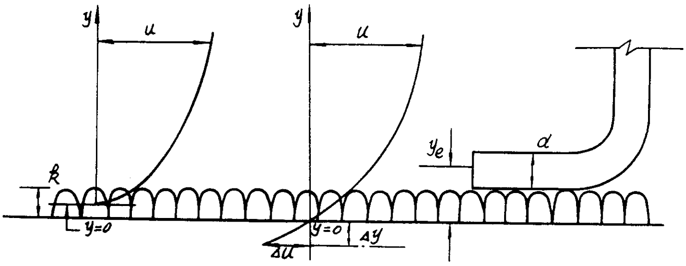


Fig. 1. Velocity profile and the Preston tube.

The calibration function of the Preston tube can be derived from Eq. 4 if the velocity profile near the surface and the displacement factor are known. In classical two-dimensional turbulent flows over smooth surfaces, the displacement factor, K_e, is a function of the Reynolds number, u_*d/υ. K_e is influenced by flows whose law of the wall deviate from the classical smooth-wall case. Instead of finding K_e versus the Reynolds number, an attempt is made to establish the displacement factor with the velocity distribution of the near wall flows. It is found that the displacement factor, K_e, can be approximated by a simple formula

$$K_e = \begin{cases} 1.3 & ;\dfrac{u_*d}{\upsilon} < 6.5 \\ \dfrac{x}{0.58x + 1.54} & ;\dfrac{u_*d}{\upsilon} \geq 6.5 \end{cases} \qquad x = \frac{du^+}{d(\ln y^+)}\left(y^+ = \frac{u_*d}{2\upsilon}\right) \tag{5}$$

in which x represents the gradient of the velocity profile in the center of the tube.

Figure 2 shows K_e derived from Patel's experimental results and McMillan's experimental data (McMillan 1956), along with the curve by Eq. 5.

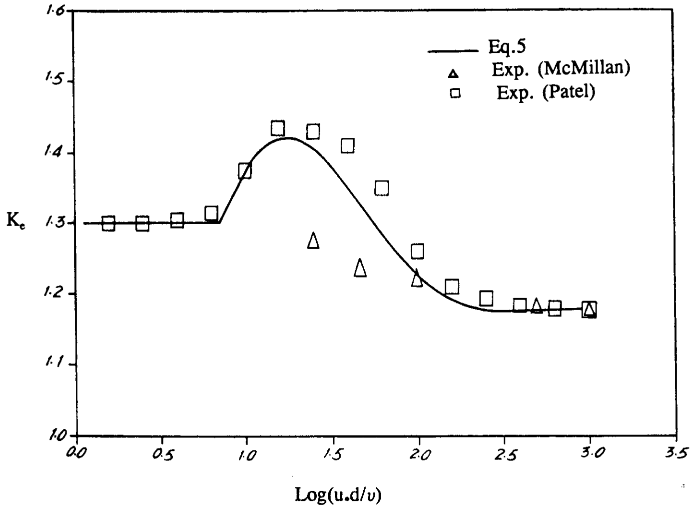


Fig. 2. Displacement factor of the Preston tube.

The velocity profile in the inner region, such as Von Dreist and Spalding's, provides a single expression for the variation of u^+ with y^+ throughout the viscous sublayer, the buffer zone, and the fully turbulent inertial zone on smooth surfaces. For the flows over rough surfaces, the most common practice uses Nikuradse's expression for completely rough turbulent flow in pipes and an equivalent sand roughness for any type of rough surfaces. Although the concept of equivalent sand roughness has been useful in the context of gross flow channel resistance computation, it has not worked satisfactorily for prediction of velocity profile for very near wall.

A Rotta (1962) model for velocity profile in two-dimensional turbulent flows over rough surfaces is shown in Fig. 1(b). According to Rotta's concept, the action of the roughness can be interpreted as being equivalent to a reduction of the viscous sublayer. It can be represented by a shift of the velocity over a smooth boundary so as to result in similar velocity profiles for both smooth and rough walls. Thus the velocity distribution is given by

$$u^{+'} = f\left(\frac{(y+\Delta y)\,u_*}{\upsilon}\right) - f\left(\frac{\Delta y u_*}{\upsilon}\right) \tag{6}$$

where Δy is the shift of the reference plane corresponding to the reduction of the velocity. The function, f, in Eq. 6 is the common law of the wall in Eq. 1. Combination of Rotta's model with Von Driest's inner region law results in a velocity profile of turbulent flow over smooth and rough surfaces (Coleman and Alonso, 1983):

$$u^+ = \int_0^{y^+} \frac{2}{1+\left(1+[2\kappa(t+\Delta y^+)]^2\left[1-\exp\left(\frac{-t-\Delta y^+}{A}\right)\right]^2\right)^{1/2}}dt \tag{7}$$

in which t is a dummy variable, $\Delta y^+ = \Delta y u_*/\upsilon$ is a roughness parameter and the Δy is the shift according to Rotta, κ is the Karman constant ($\kappa = 0.4$), A is the Driest constant ($A = 26$).

Equation 7 is parametric in that a value of u^+ can be calculated for a given value of y^+ if Δy^+ is specified. For the sand grain roughness, Δy was suggested by Cebeci and Chang (1978):

$$\frac{\Delta y}{k} = 0.9\left[(k^+)^{-1/2} - \exp\left(-\frac{k^+}{6}\right)\right] \cong \frac{0.9}{(k^+)^{0.5}} \tag{8}$$

It is found that, for the roughness of commercial pipes where the Colebrook resistance law holds true, Δy can be expressed as

$$\frac{\Delta y}{k} = \frac{1.112}{(k^+)^{0.51}} \tag{9}$$

The shift for a specified roughness with a characteristic roughness height, k, can be given by

$$\frac{\Delta y}{k} = \frac{a}{(k^+)^b}; \qquad k^+ \geq 5 \tag{10}$$

Applying Eqs. 4, 5 and 7 results in the analytical calibration function, or alternatively the analytical calibration curves of the Preston tube for surfaces of smooth, transition, and completely rough regimes. Consequently, the derived calibration function has a form of

$$\frac{\tau_0 d^2}{4\rho\upsilon^2} = F_1\left(\frac{\Delta p d^2}{4\rho\upsilon^2}, \frac{k}{d}, \frac{\Delta y}{d}\right) \tag{11}$$

As the roughness height, k, approaches zero, Eq. 11 defines a calibration function for smooth surfaces as Eq. 2. The line in Fig. 3 is the computed calibration curve for smooth surfaces together with the Patel's (1965) experimental data.

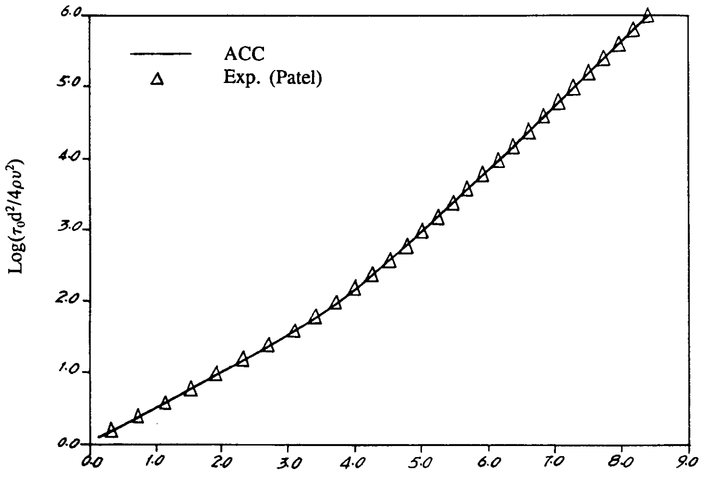
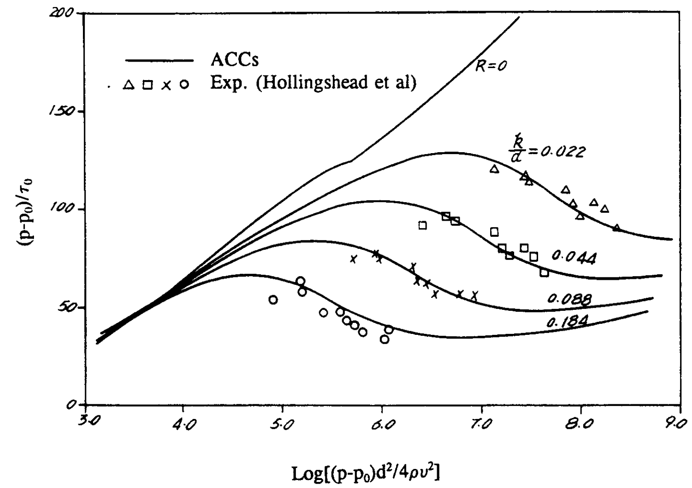
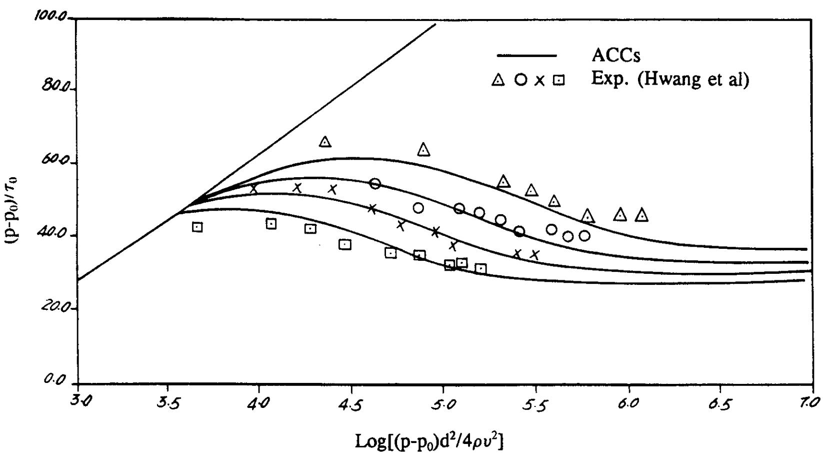


Fig. 3. Analytical calibration curve of the Preston tube (for smooth surfaces).

For a rough surface with characteristic roughness height, k, the calibration function depends on the roughness parameter, $\Delta y/k$. The analytical calibration curves can be obtained when a value of $\Delta y/k$, or alternatively the constants a and b in Eq. 10, are specified. In Fig. 4(a), the computed analytical calibration curves with the constants a and b equal to 0.50 and 0.20, respectively, are compared with some experimental date (Hollingshead and Rajaratnam 1980, Series No. 2 with sand paper roughness) in the same values of k/d. In every set of the experiments, the data covers a range from transition regime to completely rough regime. In Fig. 4(b), the computed analytical calibration curves with the constants, a and b, equal to 0.47 and 0.22, respectively, are compared with a set of Hwang and Laursen's (1963) experimental data in the transition regime. The results show that the analytical calibration function can be formulated well if the roughness parameter, $\Delta y/k$ is specified correctly.



(a)



(b)

Fig. 4. Analytical calibration curves of the Preston tube (for rough surfaces).

3 Two-tube method

Difficulties are faced by anyone using the Preston tube technique for measuring the local shear stress on a rough surface. Additional parameter(s) relating to the surface condition, the shift, Δy described in this paper, the equivalent sand roughness height, and zero datum in the previous studies, must be predetermined before the analytical calibration function can be used. When using the conventional Preston tube method, one needs to determine the roughness parameter(s) by other methods, such as measuring velocity distribution, and this procedure may counteract the simplicity and practicability of the Preston method. On the other hand, the use of the Preston tube may be not necessary if the velocity profile is measured.

One of the characteristics of the Preston tube technique is that the diameter of the tube can be chosen freely within the range of validity of the inner-layer law in turbulent shear flows, i.e. identical local shear stress should be measured using tubes of different diameters. The two-tube method is based on this principle. Instead of using one Preston tube, two tubes with different external diameters, d_1 and d_2, are used to record two dynamic pressures. A value of the local shear stress is determined from the analytical calibration function according to measured dynamic pressure by tube 1. The same procedure for tube 2 results in another local shear stress, i.e. according to Eq. 11:

$$\frac{(\tau_0)_1 d_1^2}{4\rho\upsilon^2} = F_1\left(\frac{\Delta p_1 d_1^2}{4\rho\upsilon^2}, \frac{k}{d_1}, \frac{\Delta y}{k}\right) \tag{12}$$

$$\frac{(\tau_0)_2 d_2^2}{4\rho\upsilon^2} = F_1\left(\frac{\Delta p_2 d_2^2}{4\rho\upsilon^2}, \frac{k}{d_2}, \frac{\Delta y}{k}\right) \tag{13}$$

in which $(\tau_0)_1$ and $(\tau_0)_2$ are the shear stress determined from tube 1 and tube 2, respectively. Because the analytical calibration function, F_1 in Eq. 11, and consequently the computed $(\tau_0)_1$ and $(\tau_0)_2$, depend on the assumed value of the roughness parameter, $\Delta y/k$, it is expected that an identical and correct value of the shear stress, $(\tau_0)_1 = (\tau_0)_2$, can be found by choosing values of $\Delta y/k$ iteratively. A formula for the roughness parameter can be derived from Eqs. 12 and 13:

$$F_1\left[p_1{}^*, \frac{k}{d_1}, \frac{\Delta y}{k}\right] = F_1\left[K_d^2\left(\frac{\Delta p_2}{\Delta p_1}\right)p_1{}^*, \frac{k}{d_1}\frac{1}{K_d}, \frac{\Delta y}{k}\right] / K_d^2 \tag{14}$$

in which $K_d = d_2/d_1$, $p_1{}^* = \Delta p_1 d_1^2/4\rho\upsilon^2$. The left hand of Eq. 14 is $(\tau_0{}^*)_1 = \tau_0 d_1^2/4\rho\upsilon^2$ computed from measurement of the tube 1, and the right hand of Eq. 14 is that computed from the measurement of the tube 2. Knowing d_1, d_2, Δp_1, Δp_2, and k, one can find an unique Δy, and consequently the boundary shear stress τ_0, from Eq.14. This combination of measurements by two tubes and iterative computation to find the unknown value of the roughness parameter and boundary shear stress makes it possible to measure the local shear stress in a simple way.

4 Two-tube method used on smooth surfaces

Generally the Preston tube is a total head tube. Determining the local shear stress needs pressure readings of both the total head tube and static tapping. The use of the two-tube method can eliminate the necessity of measuring the static pressure simultaneously. Suppose that p_1 and p_2 are the total pressure readings from two tubes of diameters d_1 and d_2, and p_0 is the unknown static pressure on the surface. According to the calibration function defined by Eq. 2,

$$\frac{(p_1 - p_0)\, d_1^2}{4\rho\upsilon^2} = F^{-1}\left(\frac{\tau_0 d_1^2}{4\rho\upsilon^2}\right) \tag{15}$$

$$\frac{(p_2 - p_0)\, d_2^2}{4\rho\upsilon^2} = F^{-1}\left(\frac{\tau_0 d_2^2}{4\rho\upsilon^2}\right) \tag{16}$$

in which F^{-1} refers to an inverse function of the calibration function, F. Equations 15 and 16 can be combined into:

$$\frac{(p_2 - p_1)\,d_1^2}{4\rho\upsilon^2} = F^{-1}\left(K_d^2\frac{\tau_0 d_1^2}{4\rho\upsilon^2}\right)/K_d^2 - F^{-1}\left(\frac{\tau_0 d_1^2}{4\rho\upsilon^2}\right) \tag{17}$$

This means that a new calibration function for the local shear stress can be derived from the original calibration function of the Preston tube with a form of

$$\frac{\tau_0 d_1^2}{4\rho\upsilon^2} = F_2\left(\frac{(p_2 - p_1)\,d_1^2}{4\rho\upsilon^2}, K_d\right) \tag{18}$$

This function can be easily obtained from the analytical calibration function of the Preston tube. Some of the computed calibration curves are shown in Fig. 5.

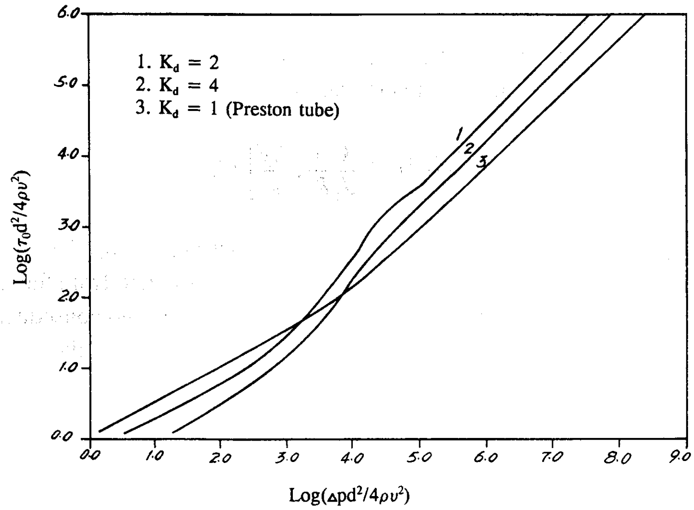


Fig. 5. Analytical calibration curves of the two-tube (for smooth surfaces).

In order to verify the proposed two-tube method used on smooth surfaces, some experimental measurements were conducted in laboratory flume. The experiment was performed in a tilting rectangular flume 0.3 m wide, 0.4 m deep, and 4 m long. The bed slope of the flume was variable and water was recirculated by a pump. Four Preston tubes with diameters, d = 1.3, 2.2, 3.9, and 6.9 mm, and ratios of inner diameter to external diameter of 0.77, 0.69, 0.64, and 0.49 were used and numbered tube 1 through 4. Various open channel conditions were obtained with different Froude number (F_r) and Reynolds number (R_e) by changing the slope of the channel and the discharge, or by using a gate installed upstream. The experimental results are listed in Table 1 in which τ_i is the shear stress determined from Preston tube i, τ_a is the averaged value of τ_0 from the four Preston tubes, τ_{ij} is that determined by two-tube method from tubes i and j. Table 2 shows the experimental conditions and the errors of each measurement result compared with the averaged value of four Preston tubes.

Table 1. Results of the Preston tubes and two tubes (τ_0:N/m^2)

Run	h (cm)	τ_a	τ_1	τ_2	τ_3	τ_4	τ_{13}	τ_{14}	τ_{24}
1	1.08	11.84	11.91	11.71	11.85	11.90	11.70	11.90	12.45
2	1.98	10.55	10.46	10.69	10.51	10.55	10.69	10.74	10.19
3	2.68	8.74	8.75	8.75	8.70	8.75	8.62	8.76	8.74
4	1.46	3.79	3.79	3.78	3.79	3.79	3.80	3.77	3.80
5	3.91	5.36	5.36	5.38	5.33	5.36	5.24	5.37	5.30
6	1.34	1.67	1.66	1.67	1.68	1.66	1.74	1.66	1.62
7	4.44	2.81	2.81	2.81	2.82	2.81	2.80	2.78	2.80
8	6.01	1.32	1.34	1.32	1.31	1.31	1.12	1.13	1.15
9	1.33	0.53	0.54	0.52	0.53	0.53	0.51	0.52	0.56
10	3.05	0.86	0.86	0.86	0.87	0.86	0.92	0.87	0.88
11	6.83	0.29	0.27	0.28	0.30	0.31	0.35	0.35	0.38

Table 2. The Experimental Conditions and Errors (%)

Run	F_r	$R_e \times 10^{-4}$	$E_{\tau 1}$	$E_{\tau 2}$	$E_{\tau 3}$	$E_{\tau 4}$	$E_{\tau 13}$	$E_{\tau 14}$	$E_{\tau 24}$
1	6.61	2.16	0.57	-1.11	0.06	0.49	-1.20	0.49	5.13
2	4.84	3.75	-0.88	1.30	-0.40	-0.24	1.30	1.78	-3.44
3	3.38	3.93	0.14	0.14	-0.43	0.14	0.24	0.26	0.03
4	2.69	1.35	0.07	-0.20	0.07	0.07	0.33	-0.46	0.33
5	2.37	4.56	0.05	0.42	-0.51	0.05	-2.19	0.23	-1.07
6	1.75	0.83	-0.45	0.15	0.75	-0.45	4.34	-0.45	-2.85
7	1.55	3.50	-0.09	-0.09	0.27	-0.09	-0.44	-1.16	-0.44
8	0.91	3.00	1.52	0.00	-0.76	-0.76	15.15	14.39	12.88
9	0.87	0.39	2.37	-1.42	0.47	0.47	-3.31	-1.42	6.16
10	0.77	1.07	-0.29	-0.29	0.87	-0.29	6.67	0.87	2.02
11	0.31	1.19	-6.90	-3.44	3.44	6.90	20.69	20.69	31.03

Although the limitation of the laboratory conditions makes it impossible to determine the local shear stress by more precise methods, such as skin friction sensors, and compare with those determined by the two-tube method, the fact that nearly identical values of the shear stress were obtained from four Preston tubes, as shown in Table 1 and Table 2, indicates that the Preston tube technique is reliable and the results from the Preston tubes could be used to compare with the results from the two tube method. The errors for the two tube method are mostly below 5 percent. It can be concluded that the differences between the Preston tube and the two-tube method are small for most flow situations. In Table 1, only the results of the two-tube method with K_d greater than two are listed. Since the precision of the method mainly depends on the difference of total pressure readings, $\Delta p = p_2 - p_1$, use of tubes with too small difference of the diameters ($K_d < 2.0$) may result in a great error because the relative error of Δp may be very large in this case. From this point of view, it is desirable to use two tubes with as great difference of the diameters as possible provided that the tubes are in the range of validality of the law of the wall, and use of two-tube method in a high velocity flow situation would have better accuracy than in a lower velocity flow situation.

5 Conclusions

The analytical calibration functions of the Preston tube for measuring the local shear stress on smooth and rough surfaces are derived based on the Rotta's model for the law of the wall over rough boundaries and a formula for displacement factor of the Preston tube. The derived calibration curve for smooth surfaces is in excellent agreement with Patel's experimental data. The analytical calibration curves for the transition and the completely rough regimes are also in agreement with experimental data when the shift Δy is appropriately specified.

Because of the simple construction and practicality of the Preston tube, it is desirable to extend the technique to rough surfaces. In order to overcome the difficulties using the Preston tube to a rough surface, a two-tube method is proposed in which measurements by two tubes of different diameters and iterative computations according to the analytical calibration functions are incorporated to determine the unknown roughness parameter. This method determines the shear stress on rough surfaces in a relatively simple way. The developed two-tube method can be also used on smooth surfaces. In this case it eliminates the necessity of measuring the static pressure simultaneously and make the measurement more easy. The two-tube method used in the smooth surfaces situation is verified by experiments.

6 Acknowledgement

The experiments were performed in Hydraulics Laboratory of Hohai University, China. The author is grateful to Mr. He Jan-jing for performing the experiments.

Notations

d, d_1, d_2	external diameter of Preston tube;
h	water depth of channel flows;
f	function of the law of the wall;
F	calibration functional of the Preston tube;
k	boundary roughness height;
k^+	scaled boundary roughness;
K_d	ratio of diameters of two tubes;
K_e	displacement factor;
p	total or static pressure (with suffixes);
R	hydraulic radius;
u	local velocity;
u^+	scaled local velocity;
u_*	shear velocity at the boundary;
V	average velocity in a cross-section of the channel;
y	distance from the boundary;
y^+	scaled distance from the boundary;
y_e	distance of the effective center of Preston tube;
κ	Karman constant;
υ	kinematic fluid viscosity;
ρ	fluid mass density;
τ_0	local shear stress;

References/Biblographie

CEBECI T., and CHANG K. C. (1978), Calculation of incompressible rough wall boundary-layer flows, J. American Institute of Aeronautics and Astronautics, 16(7), 730–735.

COLEMAN N. L. and ALONSO C. V.(1983), Two-dimensional channel flows over rough surfaces, J. Hydr. Engrg., ASCE, 109(2), 175–188.

GHOSH S. N., and ROY N. (1970), Boundary shear distribution in open channel flow, J. Hydr. Div., ASCE, 96(4), 967–993.

HOLLICK, M. (1976), Boundary shear stress measurement by Preston tube, J. Hydr. Div., ASCE, 102(7), 1053-1057.

HOLLINGSHEAD, A. B. and RAJARATNAM, N. (1980), A calibration chart for the Preston tube, J. Hydraulic Research, IAHR, 18(4) 313–326.

HWANG LI-SAN and LAURSEN E. M. (1963), Shear measurement technique for rough surfaces, J. Hydr. Div., ASCE, 89(2), 19–37.

MCMILLAN, F. A. (1956), Experiments on Pitot tubes in shear flow ARC R&M 3028.

NITSCHE, W., THUNKER, R. and HABERLAND, C. (1985), A computational Preston tube method, in Turbulent Shear Flows 4, Ed: Bradbury,L.J.S. et al, Springer-Verlag, Berlin Heideberg.

PATEL, V. C. (1965), Calibration of the Preston tube and limitations on its use in pressure gradients, J. Fluid Mech. 23, part 1, 185–208.

PRESTON, J. H. (1954), The determination of turbulent skin friction by means of Pitot tubes, J.Roy.Aeronaut. Soc.,58, 109–121.

ROTTA, J. (1962), Turbulent boundary layers in incompressible flow, Prog. in Aerospace Sci., 2, 1–219.